Network bandwidth allocation via distributed auctions with time reservations

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***Abstract*—This paper studies the problem of allocating network capacity through periodic auctions. We impose the following conditions: fully distributed solutions over an arbitrary network topology, and the requirement that resources allocated in a given auction are reserved for the entire duration of the connection, not subject to future contention. Under these conditions, we study the problem of selling capacity to optimize revenue for the operator.**

We first study optimal revenue for a single distributed auction in a general network. Next, the periodic auctions case is considered for a single link, modelling the optimal revenue problem as a Markov Decision Process (MDP); we develop a sequence of receding horizon approximations to its solution. Combining the two approaches we formulate a receding horizon optimization of revenue over a general network topology, that yields a distributed implementation. The proposal is demonstrated through simulations.

I. Introduction

The possibility of auctioning bandwidth in real time has been considered by many authors [10], [6], [11], [15], [5], [16], with a variety of applications. In this paper we are primarily motivated by the Service Overlay Network (SON) architecture [7], but the results can be applied to other type of networks with QoS constraints.

Most proposals for network auctions (e.g., the PSP mechanism of [10], or the Dutch auctions of [5]) require players to place separate bids for internal resources of the network. In contrast, we wish for users to interact with the overlay network in the simplest way, with a single bid for the entire end-to-end service, oblivious to the internal topology. To optimize the revenue of the operator for a single auction under these bids is formulated in Section II as an integer program, and a decentralized allocation mechanism is derived from its convex relaxation.

Another contribution of our work is the consideration of periodic auctions with inter-temporal constraints. References on multi-period auctions (e.g. [15]) allow future bidders to compete with incumbent ones, albeit giving the latter some advantage. This is not practical for many applications: for instance, a consumer bidding for video-on-demand content about 100 minutes long, in auctions every 5 minutes, will not participate if there is a risk of losing the connection close to the end of the movie. We thus impose the condition that once bandwidth has been allocated in an auction, the successful bidder has a *reservation* for the duration of the service: the operator must assume the risk of future auctions. Optimizing revenue with this risk is a stochastic dynamic optimization problem, that we formulate in Section III as a Markov decision process (MDP) [14], for the single resource case. We introduce a receding horizon approximation that is able to capture the dynamic aspect of the problem in a tractable way, and validate it by simulation.

In Section IV we study multi-period auctions for the general network case, with the reservation requirement. Incorporating the receding horizon term into the optimization, we develop a distributed computation. In Section V we discuss implementation issues and study the features of the proposed mechanism by simulation. Conclusions are given in Section VI. A more extensive version of this work is given in [2].

II. Optimal network bandwidth allocation

In this section we consider a set of users who bid for end-to-end bandwidth in fixed amounts, and the network must make a one-time decision to allocate capacity among them so as to maximize its revenue. The focus here is the network topology, and the requirement for a distributed resource allocation method.

The network has a set of links indexed by l, and a set of end-to-end routes indexed by r. R denotes the routing matrix. $c = (c_l)$ is the vector of link capacities. Associated with each route r is a class of service defined by a fixed bandwidth σ_r. This involves no loss of generality: if there are different classes of service over the same topological path, we use a different index r for each class. For each r, the network receives a set of N_r bids $b_r^{(i)}$, ordered as $b_r^{(1)} \geq b_r^{(2)} \geq \cdots \geq b_r^{(N_r)}$.

The resource allocation decision is to find which of these bids to accept, within the capacity constraints of the network, to maximize revenue. We will assume a first-price auction, users will pay their bid; see Section II-B for more discussion.

Since for each r the bids $b_r^{(i)}$ are for the same amount of bandwidth, the optimal allocation will involve the highest bids per route; if m_r is the number of accepted bids in route r, then the revenue in this route is $\sum_{i=1}^{m_r} b_r^{(i)}$. It will be convenient to rewrite this revenue as a function of $a_r = \sigma_r m_r$, the allocated rate, defining

$$U_{b_r}(a_r) := \sum_{i=1}^{a_r/\sigma_r} b_r^{(i)}. \quad (1)$$

This function is defined above for discrete values of a_r (the multiples of σ_r). It can be extended to a function of $a_r \in \mathbb{R}$, by linear interpolation. This piecewise linear function is increasing and concave in a_r, since bids are decreasing. With this notation, the overall optimal revenue problem is expressed as follows.

Problem 1 (Optimal instantaneous allocation):

$$\max \sum_r U_{b_r}(a_r) \quad (2a)$$

$$\text{s.t.} \sum_r R_{rl} a_r \le c_l \quad \forall l, \quad (2b)$$

$$a_r/\sigma_r \in \mathbb{Z}. \quad (2c)$$

A. Convex relaxation and distributed solution

Let us ignore for the moment the integer constraint in (2c); the optimization in (2a-2b) has the same form as the *network utility maximization* problem in the congestion control literature [8], [17], but now the utility represents the network revenue. Through the use of duality one can seek decentralized solutions to this convex relaxation. We summarize the method briefly.

Let $\alpha = (\alpha_l)$ be a vector of Lagrange multipliers (prices) associated with the constraints (2b), and let $q_r = \sum_l R_{rl}\alpha_l$ be the accumulated prices per route. Denote $y_l = \sum_r R_{rl} a_r$, $[\cdot]^+ = \max\{\cdot, 0\}$ and let $\gamma_l > 0$. Then the optimum of (2a-2b) can be found dynamically through the gradient projection algorithm

$$a_r := \arg\max_{a_r} [U_{b_r}(a_r) - q_r a_r], \quad (3a)$$

$$\alpha_l := [\alpha_l + \gamma_l (y_l - c_l)]^+. \quad (3b)$$

Here (3a) uses current route prices to fix a rate allocation with maximum "surplus" (utility minus a linear cost). (3b) compares the proposed allocation to link capacity and updates prices (up or down) accordingly. An equilibrium point of (3a-3b) is a saddle point of the Lagrangian

$$L(a, \alpha) = \sum_r U_{b_r}(a_r) + \alpha^T (c - Ra)$$

which corresponds to an optimizing a. In congestion control [17], (3) describes the adaptation of rate in the *data plane*, as a function of congestion prices generated at links. Here it corresponds to an iteration in the *control plane*, run prior to any allocation of resources.

One issue is that U_{b_r} is not strictly concave, it is piecewise linear, changing slope at the multiples of σ_r. So (3a) might have multiple optima; while this freedom allows one to impose the integer constraint (2c), it is not obvious that the algorithm would converge with this choice. If it did, we would conclude that the convex relaxation is exact. Unfortunately, this is not the case.

Example 1: Consider 4 links with capacity $c_l = 2$, and 5 paths (each with bandwidth requirement $\sigma_r = 1$), routing matrix

$$R = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

Bids for the same route are all equal, with the following distribution among routes: $b_1 = b_2 = b_3 = b_4 = 1$, and $1 < b_5 < \frac{4}{3}$. Then, it can be shown (see [2]) that the relaxed convex program (2a-2b) has solution $a^* = (\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}, 0)^T$, with optimum revenue $U^* = \frac{8}{3}$. However, adding the integer constraints (2c), the solution is $\tilde{a} = (0, 0, 0, 0, 2)^T$, with optimal integer revenue $\tilde{U} = 2b_5 < \frac{8}{3}$. So the optimal integer solution does not solve the relaxed problem, nor is it obtained by roundoff of the relaxed solution.

The above example shows that optimal revenue is not an easy integer program, its convex relaxation is not exact[1]. This forces us to accept sub-optimal allocations.

B. Strategic and game considerations

Many references in the auction literature (e.g. [10], [11], [5]) propose incentive compatible auctions, generalizing the Vickrey second-price auction [18], where the dominant user strategy is to bid the true utility. The motivation is to achieve the optimal welfare allocation. There is, however, a large complexity cost associated with implementing such truth revealing mechanisms over a network, as argued recently in [12]; in particular it implies solving a number of optimal allocation problems of the order of the number of users.

If the objective is to optimize operator revenue, this complexity is not justified: the *Revenue Equivalence Theorem* (see [4]) states that under under certain assumptions (mainly, risk neutrality of participants) all auctions have the same expected revenue for the seller. Furthermore, under other conditions (e.g., risk-averse buyers) the second-price mechanism can be *detrimental* to revenue, as the seller must subsidize truth revelation.

For these reasons we have favored a first-price auction. We refer to [2] for a more extensive discussion.

III. Periodic auctions for one link

In this section auctions are held periodically, based on bids collected for a period of length T. Once allocated, resources are reserved for a service duration that typically exceeds T, and *reservations* are in place, so that future bids are not allowed to displace incumbent users. We seek an allocation policy that maximizes revenue over time. In this section we study the auction of a single link of capacity C, with a single class of service of bandwidth $\sigma = 1$. The discrete time index k defines the auction at time kT, involving the ordered bids $b^{k,(i)}$.

[1]This limitation also occurs in [13], under opposite conditions: fixed demand, minimizing a convex cost subject to integer constraints.

If a^k represents the admitted rate (in this case equal to the number of admitted connections m^k), the associated revenue is $U_{b^k}(a^k)$, calculated as in equation (1).

A. Optimal allocation as a Markov Decision Process

We assume bids are drawn from a certain continuous probability distribution, and service durations are modeled as independent exponential random variables of mean $1/\mu$; the probability that a connection remains active at the end of a period of length T is $p := e^{-\mu T}$.

Remark 1: We assume in this section that the distribution of bids is known to the auctioneer, and their number is fixed at $N \geq C$. In Section V we will consider learning the bid distribution from past observations, and the possibility of bids arriving as a random process.

Remark 2: Duration is a characteristic of the service being auctioned. Modeling it as random and exponential is a questionable assumption, which has been adopted to allow for a Markovian analysis.

Given the distribution of bids b, we define the expected revenue function $\overline{U}(a) = E[U_b(a)]$ (increasing, piecewise linear and concave), where we replace the current bids in (1) by their expectation.

Let x^k denote the number of connections active at $t = kT^-$. The system admits a^k new connections, $0 \leq a^k \leq C - x^k$, taking the total to $x^k + a^k$. By the next auction period, $t = (k+1)T^-$, the number of active connections x^{k+1} follows then a binomial distribution with parameters $x^k + a^k$ and p.

Problem 2 (Optimal mean revenue, single link):

$$\text{Maximize} \quad \lim_n \frac{1}{n} \sum_{k=0}^{n-1} E[U_{b^k}(a^k)].$$

Here the expectation is over the vector of bids b^k and the departure process. The constraints are $0 \leq a^k \leq C - x^k$ where x^k follows the binomial transition dynamics. We can also consider the discounted version

$$\text{Maximize} \quad \sum_{k=0}^{\infty} \rho^k E[U_{b^k}(a^k)], \quad \text{where } 0 < \rho < 1.$$

Both are Markov Decision Processes (MDPs) [14]. The *state* at time k is $s_k = (x^k, b^k)$, based on which the *action* $a^k = a(s_k)$ decides on how many bids to accept. A solution to the MDP is a revenue maximizing *policy* $a(s)$. The optimal policy satisfies the Bellman equation[2]

$$V^*(x^0, b) = \max_{a \in \mathcal{A}_s} \left\{ U_b(a) + \rho E[V^*(x^1, b^1)] \right\}.$$

B. Receding horizon approximation.

Due to the difficulty of solving the Bellman equation explicitly, a commonly used approximation strategy is the *value iteration* algorithm:
$V_{m+1}(x^0, b) := \max_{a \in \mathcal{A}_s} \left\{ U_b(a) + \rho E[V_m(x^1, b^1)] \right\}.$

[2]The expectation is over the state s_1 obtained from s_0 with action $a(s_0)$. For $\rho < 1$, $V^*(\cdot)$ is the value function; this is not true for $\rho = 1$ but the characterization of the optimal $a(s)$ is still valid.

We use initial steps of the value iteration to approximate the optimal policy. Starting from $V_0 \equiv 0$, the first step is $V_1(x^0, b) = \max_{a \leq C - x^0} U_b(a) = U_b(C - x^0)$. This gives the "myopic" policy $a = C - x^0$, that sells all available capacity without regard to the future. To improve on it, we take a second value iteration step:

$$\begin{aligned} V_2(x^0, b) &= \max_{a \leq C - x^0} \{U_b(a) + \rho E[V_1(x^1, b^1)]\} \\ &= \max_{a \leq C - x^0} \{U_b(a) + \rho E[U_{b^1}(C - x^1)]\} \\ &= \max_{a \leq C - x^0} \{U_b(a) + \rho E_{x^1} \overline{U}(C - x^1)]\}. \end{aligned} \quad (4)$$

In (4), we have taken expectation with respect to the bid b^1, using $\overline{U}$ defined above; what remains is the expectation with respect to $x^1 \sim \text{Bin}(x^0 + a, p)$. The policy that solves (4) can be given a *receding horizon* interpretation: the decision optimizes over the current revenue plus the expected revenue of looking one step ahead, assuming all available capacity will be sold off at that time. This decision is then applied recursively.

Consider $W(i) = \overline{U}(C) - \overline{U}(C - i)$, piecewise linear, increasing and *convex* in i. Indeed, the increments

$$w(i) := W(i+1) - W(i) = E[b^{(C-i)}], \quad i = 1, \ldots, C$$

are non-negative and increasing in i (bids are decreasing). These properties also hold (see proof in [2]) after expectation with respect to the binomial distribution:

Proposition 1: Define $\overline{W}(x) = E[W(I_x)]$, where $I_x \sim \text{Bin}(x, p)$ for integer x, and extend by linear interpolation. Then $\overline{W}(x)$ is increasing and convex.

The optimization (4) can now be rewritten as

$$\max_{a \leq C - x^0} U_b(a) - \rho \overline{W}(x^0 + a) + \rho \overline{U}(C). \quad (5)$$

Implicit in (4) and (5) is that a is an integer. In this case, however, the condition can be relaxed without loss of generality, treating (5) as a convex optimization problem. To solve it amounts to looking for a crossing point between the derivatives of $U_b(a)$ and $\rho \overline{W}(x^0 + a)$ (marginal utilities and costs). Since the bids b are random, the marginal utility and cost curves will almost surely cross at a single, integer point.

The optimal acceptance policy is the value a such that

$$b^{(1)} \geq \cdots \geq b^{(a)} \geq \rho \overline{w}(i) > b^{(a+1)}, \quad \text{for } i = x^0 + a.$$

The values $\rho \overline{w}(i)$ act as successive *thresholds*: to accept a bids, the *lowest* one must exceed $\rho \overline{w}(x^0 + a)$.

A concrete formula for the thresholds as a function of the bid distribution is given (for the proof see [2]) by

$$\overline{w}(i) = p \sum_{l=0}^{i-1} E(b^{(C-l)}) \binom{i-1}{l} p^l (1-p)^{i-1-l}. \quad (6)$$

Based on knowledge of ρ, p, and the distribution of bids, this expression could be calculated offline and used for carrying out auctions with the policy (4). In [1] the receding horizon policy is compared with the optimal infinite-horizon MDP, in the case of one circuit ($C = 1$), where the later can be computed numerically. Results show very similar performance.

C. A fluid approximation

The stochastic calculations involved in (6) appear difficult to generalize to the network case, so we adopt a second approximation, replacing $\overline{W}(x)$ in (5) by

$$\phi(x) = W(E[I_x]) \quad \text{for } I_x \sim \text{Bin}(x, p).$$

Since $W(\cdot)$ is convex, this underestimates the one-step cost from before, $\phi(x) \leq \overline{W}(x)$. Nevertheless, if C is large the binomial distribution will be concentrated around its mean and the error is moderate. The function $\phi(x) = W(px) = \overline{U}(C) - \overline{U}(C - px)$ is still piecewise linear and convex. The fluid approximation of (5) is

$$\max_{a \leq C - x^0} U_b(a) - \rho\phi(x^0 + a) + \rho\bar{U}(C). \tag{7}$$

We can also write (7) in the equivalent form

$$\begin{aligned} &\max U_b(a) + \rho\overline{U}(z), \\ &\text{s.t. } x^0 + a \leq C, \quad p(x^0 + a) + z \leq C. \end{aligned} \tag{8}$$

At the optimum, the slack variable z satisfies its constraint with equality, $z = C - p(x^0 + a) = C - E[x^1]$, expected future allocation. Note also that interchanging $\overline{U}$ with the expectation in (4) leads directly to (8).

IV. Periodic auctions in the network case

In this section we consider the full problem of periodic auctions carried out over a general network, with the reservation requirement. In this case, we wish to incorporate future revenue through a receding horizon policy.

We describe the allocation decision at time $k = 0$. Define column vectors x^0, a, and z, whose coordinates per route r denote respectively the rate x^0_r from previous occupation, the rate allocation a_r at the current auction, and the expected rate allocation z_r in the following auction ($t = T$). We define $U_{b_r}(a_r)$ as in (1), and $\overline{U}_r(z_r)$ replacing bids by their expectation. Both are in terms of σ_r, the bandwidth requirement of the class of service associated with r. Let p_r be the probability that a connection active at $t = 0$ on route r will remain active at $t = T$, and let $P = \text{diag}(p_r)$. Then the expected input rate vector at time $t = T^-$ will be given by $P(a + x^0)$.

Problem 3 (Network receding horizon allocation):

$$\max \sum_r U_{b_r}(a_r) + \rho\overline{U}_r(z_r), \tag{9a}$$

$$\text{s.t. } R(a + x^0) \leq c, \; RP(a + x^0) + Rz \leq c, \tag{9b}$$

$$a_r/\sigma_r \in \mathbb{Z}. \tag{9c}$$

Remark 3: The above problem can also be written (see [2]) as the optimization of aggregate network utility minus a convex cost $\phi(x)$ as in (7); however $\phi(x)$ is not separable over the route components x_r, so this version does not easily lead to a distributed solution.

The convex relaxation of (9) is not exact [2]; still, we can use it to obtain an approximation. The Lagrangian $L(a, z, \alpha, \beta)$ can be expressed as

$$\begin{aligned} L = &\sum_r [U_r(a_r) - (q_r + p_r v_r)a_r] + [\rho\overline{U}_r(z_r) - v_r z_r] \\ &+ \alpha^T(c - Rx^0) + \beta^T(c - RPx^0), \end{aligned}$$

where α and β are the vectors of Lagrange multipliers (prices) for each of the constraints (9b), and $q = R^T\alpha$, $v = R^T\beta$ are the aggregate prices per route. The optimum can be found dynamically through the gradient projection algorithm which in this case takes the form

$$a_r := \arg\max_{a_r} [U_{b_r}(a_r) - (q_r + p_r v_r)a_r]; \tag{10a}$$

$$z_r := \arg\max_{z_r} [\rho\overline{U}_r(z_r) - v_r z_r]; \tag{10b}$$

$$\alpha := [\alpha + \gamma\left(R(a + x^0) - c\right)]^+; \tag{10c}$$

$$\beta := [\beta + \gamma\left(RP(a + x^0) + Rz - c\right)]^+. \tag{10d}$$

The above algorithm is similar to the one in Section II but with additional price and rate variables. (10a) compares the bids with the threshold price $q_r + p_r v_r$; (10b) involves the expected bids, and the price v_r/ρ.

The computation still inherits some difficulties of Section II: imposing integer constraints on a_r/σ_r might not yield an equilibrium, but a suboptimal allocation can be found by rounding off a_r in the decreasing direction.

V. Implementation and simulations

Implementing the described allocation algorithm in a real network should be possible with variants of current network protocols, such as RSVP. User bids are received by the brokers, with each broker associated to a service and a route, and collected until auction time. The auction allocation is then performed following the decentralized design of (10), running in the network elements. The rate reservation variables (a_r, z_r) are sent by brokers in RSVP Path messages; prices are accumulated along a path with RSVP Resv messages in the reverse direction. For more details see [2].

An important implementation issue is that the distribution of bids may not be known to the broker. In that case, we use a learning method that estimates the function $\overline{U}$ from past bids, through an exponential smoothing of the instantaneous utility function. Note that only the values of $\overline{U}$ at multiples of the circuit rate need to be updated. Moreover, the iteration applies even if the number of received bids is randomly varying in time, for instance as a stationary random process (e.g. Poisson). Here $\overline{U}(\cdot)$ is well defined, but difficult to write explicitly. Through this smoothing, we can estimate $\overline{U}$, extending the allocation method to arbitrary bid distributions and arrival patterns.

To evaluate the proposed algorithm, we implemented a discrete event simulator in JAVA. We compared the proposed policies in different network topologies, with the receding horizon achieving greater revenue with almost the same overhead calculations. We present two simulation scenarios and refer to [1], [2] for more results.

A. Scenario 1: Linear network

We simulated the topology of Fig. 1. The bids arrive as a Poisson process of intensity λ and the one-step-ahead policy based on learning of $\bar{U}$ is applied.

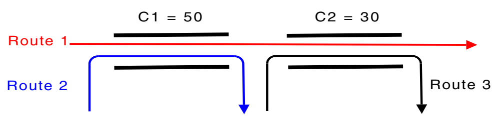

Fig. 1. Linear network with varying bids.

We compared this policy with the myopic one, keeping the time between auctions fixed at $T = 5$ min, and varying the bid arrival rate λ in every link. As for the bid distribution, the users in the long route 1 are should pay more to be allocated resources, since each of its circuits traverses 2 links. So for Fig. 2 we fixed the mean bid of route 1 to be twice that of shorter routes. The average income per unit time is displayed. We also tested this network with varying mean bid for route 1 (see [2]) .

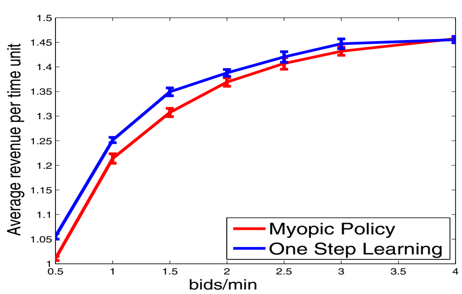

Fig. 2. Linear network with varying bid arrival rate.

B. Scenario 2: Overlay network.

Here we tested our proposal in the more realistic situation depicted in Figure 3. In this case we have four interconnected servers and several brokers. We have two types of demands: the short routes 1 and 3 use a "premium service" with 2 units of bandwidth per connection, the rest consume 1 unit. We also assume that premium demand is less frequent (20%) but its mean bid is twice the bids of shorter routes.

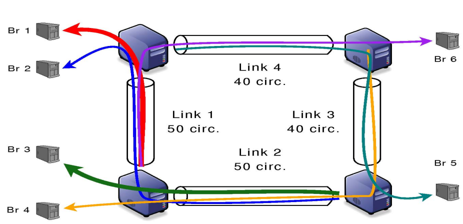

Fig. 3. Overlay Network Example.

The results are shown in Table I. We can see that the premium users who only use one link receive a substantial portion of the resources.

TABLE I
SIMULATION RESULTS FOR SCENARIO 3

Broker	1	2	3	4	5	6
Links	1	1-2	2	2-3	3-4	4-1
R_r	0.111	0.081	0.115	0.204	0.420	0.211
a_r	30.8	4.5	30.6	11.9	25.3	11.8

· R_r: revenue per unit time generated by route r.
· a_r: mean allocated rate in route r.

VI. CONCLUSIONS

In this work we studied periodic auctions for network capacity allocation with reservations. We found near-optimal policies in terms of revenue, computed via distributed convex optimization. By simulation we see that our receding horizon algorithm scales well in different network topologies, outperforms the myopic policy, and extends by on-line estimation to unknown bid distributions and arrival patterns.

In future work we will study other approximations to regularize the integer program, multiple-step extensions to the receding horizon, and the multi-path case.

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